

THE AXIOM OF CHOICE AND MAXIMAL δ -SEPARATED SETS

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ABSTRACT. We say that a subset Y of a pseudometric space (X, d) is δ -**separated** set if $d(x, y) > \delta$ for all distinct points $x, y \in Y$. The maximal δ -separated sets are widely used in the analysis on metric spaces and their existence is guaranteed by Zorn's Lemma.

In this talk we shall consider an existence of maximal δ -separated sets in metric and pseudometric spaces from the point of view the Axiom of Choice and its weaker forms. We shall also indicate a set-theoretic status of the well-known theorem whose known proofs are based on Vitali $5r$ -covering lemma or maximal δ -separated sets:

Theorem. Let (X, d) be a pseudometric space. Then, the space X is separable if and only if there exists a Borel measure μ on X such that the measure of every open ball is positive and finite.

These are joint results with Przemysław Górką.